УДК 51.330.115

THE EXISTENCE OF EQUILIBRIUM IN THE NONLINEAR VON NEUMANN MODEL

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A nonlinear von Neumann model is defined by two pairs (A_1, B_1) and (A_2, B_2) of real valued functions defined on the product $X \times Y$ of two sets X, Y and such that Kemeny-Morgenstern-Thompson'conditions (KMT'conditions) are satisfied:

$$\forall x \in X \quad \exists y \in Y \; A_1(x,y) > 0$$

$$\forall y \in Y \quad \exists x \in X \; B_2(x,y) > 0.$$

We say that a pair of points $(\bar{x}, \bar{y})$ is an equilibrium, if there exists numbers $\lambda > 0$, $\mu > 0$ such that

$$\forall y \in Y \quad \lambda A_1(\bar{x}, y) \leq B_1(\bar{x}, y)$$
$$\forall x \in X \quad B_2(x, \bar{y}) \leq \mu A_2(x, \bar{y})$$
$$\lambda A_1(\bar{x}, \bar{y}) = B_1(\bar{x}, \bar{y})$$
$$B_2(\bar{x}, \bar{y}) = \mu A_2(\bar{x}, \bar{y})$$
$$B_1(\bar{x}, \bar{y}) > 0$$
$$A_2(\bar{x}, \bar{y}) > 0$$

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Let us observe, that in the case $\lambda=\mu$, $A_1=A_2$, $B_1=B_2$ the equilibrium defined above coincides with the equilibrium in the two functions nonlinear von Neumann model (see [2]).

Analogously as in [2] we shall assume some additional restrictions on X, Y and A_1, B_1, A_2, B_2 to ensure that the nonlinear von Neumann model has an equilibrium:

(i) X, Y are compact, convex subsets of Hausdorff local convex spaces E_1 and E_2, respectively;

(ii) A_1, B_1, A_2, B_2 are continuous functions and, A_1 (B_1) is concave (convex) in y for every fixed x and A_2 (B_2) is convex (concave) in x for every fixed y;

(iii) for each $(x,y) \in X \times Y$ $B_1(x,y)>0$ and $A_2(x,y)>0$.

THEOREM. Under assumptions (i) - (iii) the nonlinear von Neumann model has an equilibrium.

P r o o f . By (iii) the conditions (5), (5') are satisfied. To prove the rest of conditions we shall use the fixed point theorem of *Fan* [1] (Theorem 1, p. 122). For $x \in X$ and $y \in Y$ let us denote

$$V(x)=\{y' \in X \mid \exists_{0<\alpha<1} \max_y [(1-\alpha)A_1(x,y)-\alpha B_1(x,y)] =$$
$$=(1-\alpha)A_1(x,y')-\alpha B_1(x,y')=0\}$$

$$W(y)=\{x' \in X \mid \exists_{0<\beta<1} \min_x [(1-\beta)A_2(x,y)-\beta B_2(x,y)] =$$
$$=(1-\beta)A_2(x',y)-\beta B_2(x',y)=0\}.$$

By (ii) and (iii) the sets $V(x)$ and $W(y)$ are nonempty, convex and closed. Furthermore, the function $\Phi : X\times Y \to 2^{X\times Y}$ defined as $\Phi(x,y)=W(y)\times V(x)$ is upper semi-continuous. The set $X\times Y$ is compact, convex subset of Hausdorff local convex space $E_1 \times E_2$ because of (i). Thus, by Fan's theorem there exists a pair $(\bar{x},\bar{y})$ such that $(\bar{x},\bar{y})\in W(\bar{y})\times V(\bar{x})$. Now, it is easy to check that for $(\bar{x},\bar{y})$ and $\lambda=\frac{1-\alpha}{\alpha}$, $\mu=\frac{1-\beta}{\beta}$

the conditions (3), (3'), (4), (4') are fulfilled.

Q.E.D.

Similar problems in the linear von Neumann model was examined by *Łos* [3].

I am indebted to Professor J.Los for suggesting me the problem considered in this paper.

REFERENCES

1. FAN K. Fixed- point and minimax theorems in locally convex topological linear spaces. - "Proc. Nat. Acad. Sci USA", 1952, v. 38, p.121 - 126.
2. IDZIK A. Farkas lemma for concave-convex functions with an application to the nonlinear von Neumann model. - In: "Computation Centre PAS Reports", 1973 , №117, Warsaw
3. LOS J. Labour, consumption and wages in a von Neumann model.- In: "Mathematical Models in Economics (eds. J.Los and M.Los)", 1974 (North-Holland Publishing Company, Amsterdam - London).